1.12 For low-speed (laminar) flow in a tube of radius r_o, the velocity u takes the form

$$u = B\frac{\Delta p}{\mu}\left(r_o^2 - r^2\right)$$

where μ is viscosity and Δp the pressure drop. What are the dimensions of B?

Solution: Using Table 1-2, write this equation in dimensional form:

$$\{u\} = \{B\}\frac{\{\Delta p\}}{\{\mu\}}\{r^2\}, \quad \text{or:} \quad \left\{\frac{L}{T}\right\} = \{B?\}\frac{\{M/LT^2\}}{\{M/LT\}}\{L^2\} = \{B?\}\left\{\frac{L^2}{T}\right\},$$

$$\text{or:} \quad \mathbf{\{B\} = \{L^{-1}\}} \quad \textit{Ans.}$$

The parameter B must have dimensions of inverse length. In fact, B is not a constant, it hides one of the variables in pipe flow. The proper form of the pipe flow relation is

$$u = C\frac{\Delta p}{L\mu}\left(r_o^2 - r^2\right)$$

where L is the *length of the pipe* and C is a dimensionless constant which has the theoretical laminar-flow value of (1/4)—see Sect. 6.4.

1.13 The efficiency η of a pump is defined as

$$\eta = \frac{Q\Delta p}{\text{Input Power}}$$

where Q is volume flow and Δp the pressure rise produced by the pump. What is η if Δp = 35 psi, Q = 40 L/s, and the input power is 16 horsepower?

Solution: The student should perhaps verify that QΔp has units of power, so that η is a dimensionless ratio. Then convert everything to consistent units, for example, BG:

$$Q = 40\ \frac{L}{s} = 1.41\ \frac{ft^2}{s}; \quad \Delta p = 35\ \frac{lbf}{in^2} = 5040\ \frac{lbf}{ft^2}; \quad \text{Power} = 16(550) = 8800\ \frac{ft\cdot lbf}{s}$$

$$\eta = \frac{(1.41\ ft^3/s)(5040\ lbf/ft^2)}{8800\ ft\cdot lbf/s} \approx 0.81 \quad \text{or} \quad \mathbf{81\%} \quad \textit{Ans.}$$

Similarly, one could convert to SI units: Q = 0.04 m^3/s, Δp = 241300 Pa, and input power = 16(745.7) = 11930 W, thus h = (0.04)(241300)/(11930) = **0.81**. *Ans.*

For N_2O, from Table A-4, $k \approx 1.31$, so $B_{N_2O} = 1.31\text{ atm} = \mathbf{1.33E5\ Pa}$ *Ans.* (a)

For water at 20°C, we could just look it up in Table A-3, but we more usefully try to estimate B from the state relation (1-22). Thus, for a liquid, approximately,

$$B \approx \rho\frac{d}{d\rho}[p_o\{(B+1)(\rho/\rho_o)^n - B\}] = n(B+1)p_o(\rho/\rho_o)^n = n(B+1)p_o \quad \text{at 1 atm}$$

For water, $B \approx 3000$ and $n \approx 7$, so our estimate is

$$B_{water} \approx 7(3001)p_o = 21007\text{ atm} \approx \mathbf{2.13E9\ Pa} \quad \textit{Ans.}\text{ (b)}$$

This is 2.7% less than the value B = 2.19E9 Pa listed in Table A-3.

1.37 A near-ideal gas has M = 44 and c_V = 610 J/(kg·K). At 100°C, what are (a) its specific heat ratio, and (b) its speed of sound?

Solution: The gas constant is $R = \Lambda/M = 8314/44 \approx 189$ J/(kg·K). Then

$$c_v = R/(k-1), \quad \text{or:} \quad k = 1 + R/c_v = 1 + 189/610 \approx \mathbf{1.31} \quad \textit{Ans.}\text{ (a)} \quad [\text{It is probably } N_2O]$$

With k and R known, the speed of sound at 100ºC = 373 K is estimated by

$$a = \sqrt{kRT} = \sqrt{1.31[189\ m^2/(s^2\cdot K)](373\ K)} \approx \mathbf{304\ m/s} \quad \textit{Ans.}\text{ (b)}$$

1.38 In Fig. P1.38, if the fluid is glycerin at 20°C and the width between plates is 6 mm, what shear stress (in Pa) is required to move the upper plate at V = 5.5 m/s? What is the flow Reynolds number if "L" is taken to be the distance between plates?

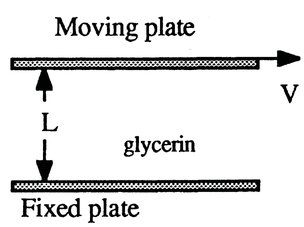


Fig. P1.38

Solution: (a) For glycerin at 20°C, from Table 1.4, $\mu \approx 1.5\ N\cdot s/m^2$. The shear stress is found from Eq. (1) of Ex. 1.8:

$$\tau = \frac{\mu V}{h} = \frac{(1.5\ Pa\cdot s)(5.5\ m/s)}{(0.006\ m)} \approx \mathbf{1380\ Pa} \quad \textit{Ans.}\text{ (a)}$$

The density of glycerin at 20°C is 1264 kg/m^3. Then the Reynolds number is defined by Eq. (1.24), with L = h, and is found to be decidedly laminar, Re < 1500:

$$Re_L = \frac{\rho VL}{\mu} = \frac{(1264\ kg/m^3)(5.5\ m/s)(0.006\ m)}{1.5\ kg/m\cdot s} \approx \mathbf{28} \quad \textit{Ans.}\text{ (b)}$$

$$\text{Least-squares of } \ln(\mu) \text{ versus } \frac{1}{T}: \quad \boldsymbol{\mu \approx 3.31E{-}9 \ \frac{kg}{m \cdot s} \ \exp\left(\frac{5476\ K}{T°K}\right)} \quad \textit{Ans. } (\#2)$$

The accuracy is somewhat better, but not great, as follows:

T, °C:	0	20	40	60	80	100
μSAE30, kg/m·s:	2.00	0.40	0.11	0.042	0.017	0.0095
Curve-fit #1:	2.00	0.42	0.108	0.033	0.011	0.0044
Curve-fit #2:	1.68	0.43	0.13	0.046	0.018	0.0078

Neither fit is worth writing home about. Andrade's equation is not accurate for SAE 30 oil.

1.45 A block of weight W slides down an inclined plane on a thin film of oil, as in Fig. P1.45 at right. The film contact area is A and its thickness h. Assuming a linear velocity distribution in the film, derive an analytic expression for the terminal velocity V of the block.

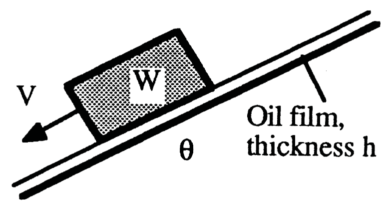


Fig. P1.45

Solution: Let "x" be down the incline, in the direction of V. By "terminal" velocity we mean that there is no acceleration. Assume a linear viscous velocity distribution in the film below the block. Then a force balance in the x direction gives:

$$\sum F_x = W \sin\theta - \tau A = W \sin\theta - \left(\mu \frac{V}{h}\right) A = m a_x = 0,$$

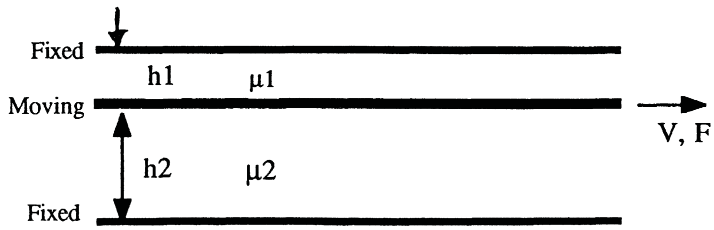
$$\text{or:} \quad V_{\text{terminal}} = \boldsymbol{\frac{hW \sin\theta}{\mu A}} \quad \textit{Ans.}$$

P1.46 A simple and popular model for two non-newtonian fluids in Fig. 1.9*a* is the *power-law*:

$$\tau \quad \approx \quad C\,(\frac{du}{dy})^n$$

where C and n are constants fit to the fluid [15]. From Fig. 1.9*a*, deduce the values of the exponent n for which the fluid is (*a*) newtonian; (*b*) dilatant; and (*c*) pseudoplastic. (*d*) Consider the specific model constant $C = 0.4$ N-s^n/m^2, with the fluid being sheared between two parallel plates as in Fig. 1.8. If the shear stress in the fluid is 1200 Pa, find the velocity V of the upper plate for the cases (*d*) $n = 1.0$; (*e*) $n = 1.2$; and (*f*) $n = 0.8$.

1.48 A thin moving plate is separated from two fixed plates by two fluids of unequal viscosity and unequal spacing, as shown below. The contact area is A. Determine (a) the force required, and (b) is there a necessary relation between the two viscosity values?



Solution: (a) Assuming a linear velocity distribution on each side of the plate, we obtain

$$F = \tau_1 A + \tau_2 A = \left(\frac{\mu_1 \mathbf{V}}{\mathbf{h}_1} + \frac{\mu_2 \mathbf{V}}{\mathbf{h}_2}\right)\mathbf{A} \quad \textit{Ans.}\ \textbf{(a)}$$

The formula is of course valid only for laminar (nonturbulent) steady viscous flow.
(b) Since the center plate separates the two fluids, they may have separate, unrelated shear stresses, and there is ***no necessary relation*** between the two viscosities.

1.49 An amazing number of commercial and laboratory devices have been developed to measure fluid viscosity, as described in Ref. 27. Consider a concentric shaft, as in Prob. 1.47, but now fixed axially and rotated inside the sleeve. Let the inner and outer cylinders have radii r_i and r_o, respectively, with total sleeve length L. Let the rotational rate be Ω (rad/s) and the applied torque be M. Using these parameters, derive a theoretical relation for the viscosity μ of the fluid between the cylinders.

Solution: Assuming a linear velocity distribution in the annular clearance, the shear stress is

$$\tau = \mu\frac{\Delta V}{\Delta r} \approx \mu\frac{\Omega r_i}{r_o - r_i}$$

This stress causes a force $dF = \tau\, dA = \tau(r_i\, d\theta)L$ on each element of surface area of the inner shaft. The moment of this force about the shaft axis is $dM = r_i\, dF$. Put all this together:

$$M = \int r_i\, dF = \int_0^{2\pi} r_i \mu \frac{\Omega r_i}{r_o - r_i} r_i\, L\, d\theta = \frac{2\pi\mu\Omega r_i^3 L}{r_o - r_i}$$

Solve for the viscosity: $\boldsymbol{\mu \approx M(r_o - r_i)/\{2\pi\Omega r_i^3 L\}}$ *Ans.*

P1.51 An approximation for the boundary-layer shape in Figs. 1.6*b* and P1.51 is the formula

$$u(y) \approx U\sin\left(\frac{\pi y}{2\delta}\right), \quad 0 \le y \le \delta$$

where U is the stream velocity far from the wall and δ is the boundary layer thickness, as in Fig. P.151. If the fluid is helium at 20°C and 1 atm, and if U = 10.8 m/s and δ = 3 mm, use the formula to (*a*) estimate the wall shear stress τ_w in Pa; and (*b*) find the position in the boundary layer where τ is one-half of τ_w.

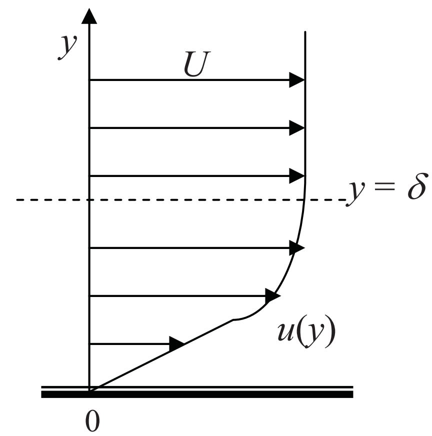


Fig. P1.51

Solution: From Table A.4, for helium, take R = 2077 m^2/(s^2-K) and μ = 1.97E-5 kg/m-s.
(*a*) Then the wall shear stress is calculated as
A very small shear stress, but it has a profound effect on the flow pattern.

$$\tau_w = \mu\frac{\partial u}{\partial y}\Big|_{y=0} = \mu\left(U\frac{\pi}{2\delta}\cos\frac{\pi y}{2\delta}\right)_{y=0} = \frac{\pi\,\mu U}{2\delta}$$

$$\text{Numerical values: } \tau_w = \frac{\pi(1.97E-5\,kg/m-s)(10.8\,m/s)}{2(0.003\,m)} = \mathbf{0.11\ Pa} \qquad Ans.(a)$$

(*b*) The variation of shear stress across the boundary layer is simply a cosine wave:

$$\tau(y) = \frac{\pi\mu U}{2\delta}\cos\left(\frac{\pi y}{2\delta}\right) = \tau_w\cos\left(\frac{\pi y}{2\delta}\right) = \frac{\tau_w}{2} \text{ when } \frac{\pi y}{2\delta} = \frac{\pi}{3}, \text{ or: } \mathbf{y} = \frac{\mathbf{2\delta}}{\mathbf{3}} \qquad Ans.(b)$$

1.52 The belt in Fig. P1.52 moves at steady velocity V and skims the top of a tank of oil of viscosity μ. Assuming a linear velocity profile, develop a simple formula for the belt-

Separating the variables, we may integrate:

$$\int_{\omega_o}^{w} \frac{d\omega}{\omega} = -\frac{\pi\mu r_o^4}{2hI_o \sin\theta}\int_0^t dt, \quad \text{or:} \quad \boldsymbol{\omega = \omega_o \exp\left[-\frac{5\pi\mu r_o^2 t}{3mh\sin\theta}\right]} \quad \textit{Ans.}$$

1.54* A disk of radius R rotates at angular velocity Ω inside an oil container of viscosity μ, as in Fig. P1.54. Assuming a linear velocity profile and neglecting shear on the outer disk edges, derive an expression for the viscous torque on the disk.

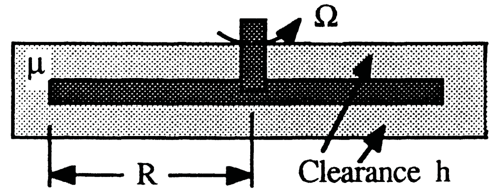


Fig. P1.54

Solution: At any r ≤ R, the viscous shear $\tau \approx \mu\Omega r/h$ on both sides of the disk. Thus,

$$d(\text{torque}) = dM = 2r\tau\, dA_w = 2r\frac{\mu\Omega r}{h}2\pi r\, dr,$$

$$\text{or:} \quad M = 4\pi\frac{\mu\Omega}{h}\int_0^R r^3\, dr = \boldsymbol{\frac{\pi\mu\Omega R^4}{h}} \quad \textit{Ans.}$$

P1.55 A block of weight W is being pulled over a table by another weight W_o, as shown in Fig. P1.55. Find an algebraic formula for the steady velocity U of the block if it slides on an oil film of thickness h and viscosity μ. The block bottom area A is in contact with the oil. Neglect the cord weight and the pulley friction.

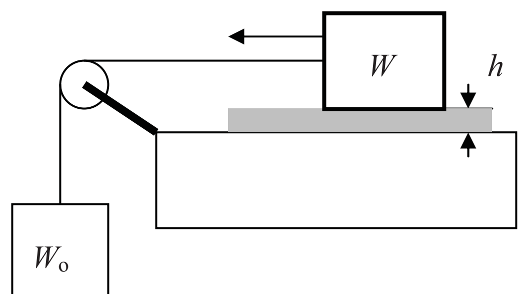


Fig. P1.55

Solution: This problem is a lot easier to *solve* than to set up and sketch. For steady motion, there is no acceleration, and the falling weight balances the viscous resistance of the oil film:

The complete (small-slope) solution to this problem is:

$$\boldsymbol{\eta = h\exp[-(\rho g/Y)^{1/2}x]}, \quad \text{where } h = (Y/\rho g)^{1/2}\cot\theta \quad \textit{Ans.}$$

The formula clearly satisfies the requirement that $\eta = 0$ if $x = \infty$. It requires "small slope" and therefore the contact angle should be in the range $70° < \theta < 110°$.

1.69 A solid cylindrical needle of diameter d, length L, and density ρ_n may "float" on a liquid surface. Neglect buoyancy and assume a contact angle of 0°. Calculate the maximum diameter needle able to float on the surface.

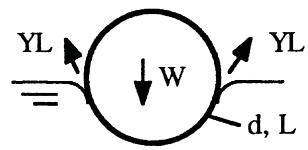


Fig. P1.69

Solution: The needle "dents" the surface downward and the surface tension forces are upward, as shown. If these tensions are nearly vertical, a vertical force balance gives:

$$\sum F_z = 0 = 2YL - \rho g\frac{\pi}{4}d^2L, \quad \text{or:} \quad \mathbf{d_{max}} \approx \sqrt{\frac{\mathbf{8Y}}{\boldsymbol{\pi\rho g}}} \quad \textit{Ans. (a)}$$

(b) Calculate d_{max} for a steel needle (SG ≈ 7.84) in water at 20°C. The formula becomes:

$$d_{max} = \sqrt{\frac{8Y}{\pi\rho g}} = \sqrt{\frac{8(0.073\text{ N/m})}{\pi(7.84\times 998\text{ kg/m}^3)(9.81\text{ m/s}^2)}} \approx 0.00156\text{ m} \approx \mathbf{1.6\ mm} \quad \textit{Ans. (b)}$$

1.70 Derive an expression for the capillary-height change h, as shown, for a fluid of surface tension Y and contact angle θ between two parallel plates W apart. Evaluate h for water at 20°C if W = 0.5 mm.

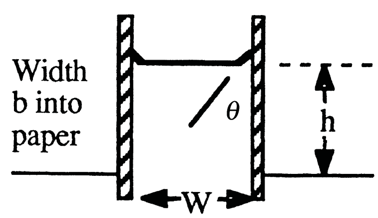


Fig. P1.70

Solution: With *b* the width of the plates into the paper, the capillary forces on each wall together balance the weight of water held above the reservoir free surface:

$$\rho gWhb = 2(Yb\cos\theta), \quad \text{or:} \quad h \approx \frac{\mathbf{2Y\cos\theta}}{\boldsymbol{\rho g W}} \quad \textit{Ans.}$$

For water at 20°C, $Y \approx 0.0728$ N/m, $\rho g \approx 9790$ N/m^3, and $\theta \approx 0°$. Thus, for W = 0.5 mm,

$$h = \frac{2(0.0728 \text{ N/m})\cos 0°}{(9790 \text{ N/m}^3)(0.0005 \text{ m})} \approx 0.030 \text{ m} \approx \mathbf{30\ mm} \quad \textit{Ans.}$$

1.71* A soap bubble of diameter D_1 coalesces with another bubble of diameter D_2 to form a single bubble D_3 with the same amount of air. For an isothermal process, express D_3 as a function of D_1, D_2, p_{atm}, and surface tension Y.

Solution: The masses remain the same for an isothermal process of an ideal gas:

$$m_1 + m_2 = \rho_1 \upsilon_1 + \rho_2 \upsilon_2 = m_3 = \rho_3 \upsilon_3,$$

$$\text{or: } \left(\frac{p_a + 4Y/r_1}{RT}\right)\left(\frac{\pi}{6}D_1^3\right) + \left(\frac{p_a + 4Y/r_2}{RT}\right)\left(\frac{\pi}{6}D_2^3\right) = \left(\frac{p_a + 4Y/r_3}{RT}\right)\left(\frac{\pi}{6}D_3^3\right)$$

The temperature cancels out, and we may clean up and rearrange as follows:

$$\mathbf{p_a D_3^3 + 8YD_3^2 = \left(p_a D_2^3 + 8YD_2^2\right) + \left(p_a D_1^3 + 8YD_1^2\right)} \quad \textit{Ans.}$$

This is a cubic polynomial with a known right hand side, to be solved for D_3.

1.72 Early mountaineers boiled water to estimate their altitude. If they reach the top and find that water boils at 84°C, approximately how high is the mountain?

Solution: From Table A-5 at 84°C, vapor pressure $p_v \approx 55.4$ kPa. We may use this value to interpolate in the standard altitude, Table A-6, to estimate

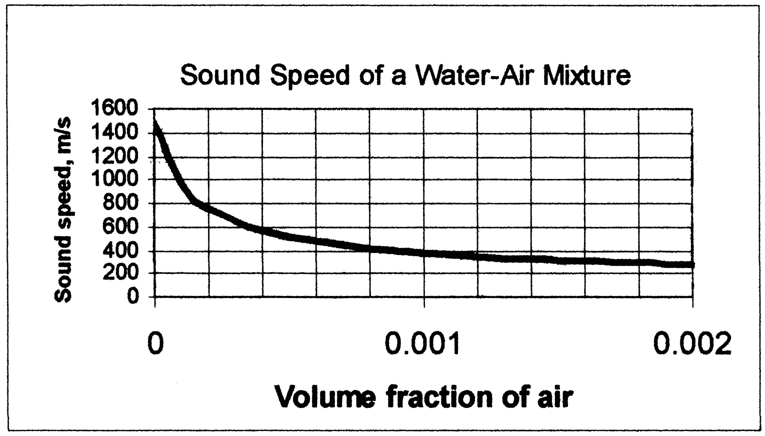
$$z \approx \mathbf{4800\ m} \quad \textit{Ans.}$$

1.73 A small submersible moves at velocity V in 20°C water at 2-m depth, where ambient pressure is 131 kPa. Its critical cavitation number is $Ca \approx 0.25$. At what velocity will cavitation bubbles form? Will the body cavitate if V = 30 m/s and the water is cold (5°C)?

Solution: From Table A-5 at 20°C read $p_v = 2.337$ kPa. By definition,

$$Ca_{crit} = 0.25 = \frac{2(p_a - p_v)}{\rho V^2} = \frac{2(131000 - 2337)}{(998 \text{ kg/m}^3)V^2}, \quad \text{solve } V_{crit} \approx \mathbf{32.1\ m/s} \quad \textit{Ans.}\text{ (a)}$$

(b) For the given data, a plot of sound speed versus gas volume fraction is as follows:



The difference in air and water compressibility is so great that the speed drop-off is quite sharp.

1.80* A two-dimensional steady velocity field is given by $u = x^2 - y^2$, $v = -2xy$. Find the streamline pattern and sketch a few lines. [*Hint*: The differential equation is exact.]

Solution: Equation (1.44) leads to the differential equation:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dx}{x^2 - y^2} = \frac{dy}{-2xy}, \quad \text{or:} \quad (2xy)dx + (x^2 - y^2)dy = 0$$

As hinted, this equation is *exact*, that is, it has the form $dF = (\partial F/\partial x)dx + (\partial F/\partial y)dy = 0$. We may check this readily by noting that $\partial/\partial y(2xy) = \partial/\partial x(x^2 - y^2) = 2x = \partial^2 F/\partial x \partial y$. Thus we may integrate to give the formula for streamlines:

$$\mathbf{F = x^2y - y^3/3 + constant} \quad \textit{Ans.}$$

This represents (inviscid) flow in a series of 60° corners, as shown in Fig. E4.7a of the text. [This flow is also discussed at length in Section 4.7.]

1.81 Repeat Ex. 1.13 by letting the velocity components increase linearly with time:

$$\mathbf{V} = Kxt\mathbf{i} - Kyt\mathbf{j} + 0\mathbf{k}$$

Solution: The flow is unsteady and two-dimensional, and Eq. (1.44) still holds:

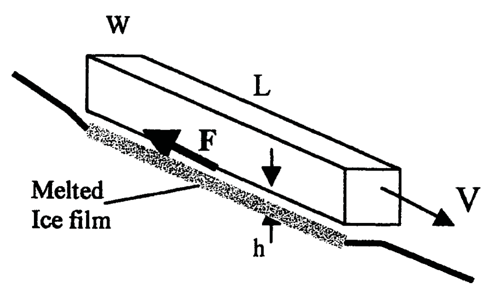
$$\text{Streamline:} \quad \frac{dx}{u} = \frac{dy}{v}, \quad \text{or:} \quad \frac{dx}{Kxt} = \frac{dy}{-Kyt}$$

C1.2 When a person ice-skates, the ice surface actually melts beneath the blades, so that he or she skates on a thin film of water between the blade and the ice. (a) Find an expression for total friction force F on the bottom of the blade as a function of skater velocity V, blade length L, water film thickness h, water viscosity μ, and blade width W. (b) Suppose a skater of mass *m*, moving at constant speed V_o, suddenly stands stiffly with skates pointed directly forward and allows herself to coast to a stop. Neglecting air resistance, how far will she travel (on *two* blades) before she stops? Give the answer X as a function of (V_o, *m*, L, h, μ, W). (c) Compute X for the case $V_o = 4$ m/s, m = 100 kg, L = 30 cm, W = 5 mm, and h = 0.1 mm. Do you think our assumption of negligible air resistance was a good one?

Solution: (a) The skate bottom and the melted ice are like two parallel plates:

$$\tau = \mu\frac{V}{h}, \quad F = \tau A = \frac{\mu VLW}{h} \quad \textit{Ans. } \text{(a)}$$

(b) Use **F** = m**a** to find the stopping distance:



$$\Sigma F_x = -F = -\frac{2\mu VLW}{h} = ma_x = m\frac{dV}{dt}$$

(*the* '2' *is for two blades*)

Separate and integrate once to find the velocity, once again to find the distance traveled:

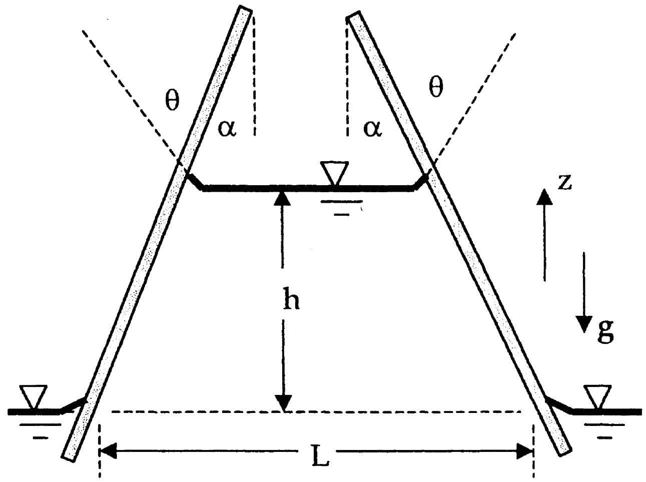
$$\int \frac{dV}{V} = -\int \frac{2\mu LW}{mh}dt, \quad or: \quad V = V_o e^{\frac{-2\mu LW}{mh}t}, \quad X = \int_0^\infty V\,dt = \frac{V_o mh}{2\mu LW} \quad \textit{Ans. } \text{(b)}$$

(c) Apply our specific numerical values to a 100-kg (!) person:

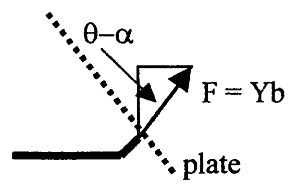
$$X = \frac{(4.0\ m/s)(100\ kg)(0.0001\ m)}{2(1.788E\text{–}3\ kg/m\cdot s)(0.3\ m)(0.005\ m)} = \mathbf{7460\ m\ (!)} \quad \textit{Ans. } \text{(c)}$$

We could coast to the next town on ice skates! It appears that our assumption of negligible air drag was grossly incorrect.

C1.3 Two thin flat plates are tilted at an angle α and placed in a tank of known surface tension Y and contact angle θ, as shown. At the free surface of the liquid in the tank, the two plates are a distance *L* apart, and of width *b* into the paper. (a) What is the total *z*-directed force, due to surface tension, acting on the liquid column between plates? (b) If the liquid density is ρ, find an expression for Y in terms of the other variables.



Solution: (a) Considering the right side of the liquid column, the surface tension acts tangent to the local surface, that is, along the dashed line at right. This force has magnitude F = Yb, as shown. Its vertical component is F $\cos(\theta-\alpha)$, as shown. There are two plates. Therefore, the total z-directed force on the liquid column is



$$\mathbf{F_{vertical} = 2Yb\cos(\theta-\alpha)} \quad \textit{Ans.}\text{ (a)}$$

(b) The vertical force in (a) above holds up the entire weight of the liquid column between plates, which is $W = \rho g\{bh(L - h\tan\alpha)\}$. Set W equal to F and solve for

$$\mathbf{U = [\rho gbh(L - h\tan\alpha)]/[2\cos(\theta-\alpha)]} \quad \textit{Ans.}\text{ (b)}$$

C1.4 Oil of viscosity μ and density ρ drains steadily down the side of a tall, wide vertical plate, as shown. The film is fully developed, that is, its thickness δ and velocity profile w(x) are independent of distance z down the plate. Assume that the atmosphere offers no shear resistance to the film surface.

(a) Sketch the approximate shape of the velocity profile w(x), keeping in mind the boundary conditions.